anything remarkable noticed in its position at any subsequent date. Its existence seems therefore to have been a very short one.

Of the two sketches forwarded, No. 128 is a portion of the ordinary eyepiece chart for the night's work. No. 128a is a copy of the sketch made after the appearance of the spot in the observing note-book, the neighbourhood being filled in from No. 128.

Trincomali, Ceylon:
1905 *April* 12.

Further Note on the Density and Prolateness of close Binary Stars. By Alex. W. Roberts, D.Sc.

In vol. lxiii. (p. 527) of the *Monthly Notices* I considered the problem of determining the orbital elements of a close binary system from the light-changes produced by the mutual eclipse of the component stars as they circled round one another.

It is indeed only from photometric observations that we may hope to determine what many would regard as two of the most important facts in cosmic physics, the density of structure and the prolateness of figure of two masses of matter revolving almost, or actually in contact, round their common centre of gravity.

Fortunately the difficulty of the problem, at least when considered generally, is not in any way a serious one. The problem consists, in its simplest form, in determining the area of the projected surface of two contiguous revolving masses. The light-changes of a close binary system are of course dependent on this ever-changing area. Given, therefore, a very accurately determined light-curve, we may readily ascertain the area included within the projected outline of the system; and this being known, it is but another step to the determination of the orbital elements, of figure, of position, of movement, which govern the form of the projected area.

This of course is only a general statement of the problem. A rigorous consideration of it is a more involved matter. We may indicate one or two of the difficulties that arise.

If the orbit of a close binary system be at all eccentric, and the density small, as is usually the case with stars of this class, then there will be a constant change of figure as the mutual attraction varies in intensity. To give a concrete example, the major axis of β *Lyræ* varies in length every revolution, through at least one million miles, and this contraction and expansion along the major axis, and commensurate expansion and contraction along the two axes at right angles to it, is a continuous movement. Further, the turning points of this oscillation do not coincide with the apsidal line, but are modified by the density

and rigidity of the system, that is, by its power to adapt itself readily to the ever-changing forces acting upon it.

Then again this constant change of figure of such systems as are eccentric in orbit must produce a constant alteration in the surface pressure of the system, and this variation will naturally display itself in minute yet measurable light-changes.

These subordinate fluctuations of figure, pressure, light, and heat, are represented in every carefully determined light or motion curve. They appear as some of the contradictions and aberrations which either the spectroscope or the photometer reveals to us with regard to such stars as *β Lyræ*, *V Puppis*, and *RR Centauri*.

And in this connection I would strongly urge a constant watch being kept on stars of this class, for while the *mean* light-curve of a close binary star remains unchanged, each group of observations yields departures from this mean curve that are significant and important.

Every observer who has made extensive photometric observations of any single Algol binary star knows how dissimilar in points of detail, yet similar in their general family type, are the various curves obtained by grouping observations in sets. To give again a concrete example, let the light-curves of *β Lyræ* as determined by Goodrick (*De stella β Lyræ variabile*, p. 22); by Argelander (*De stella β Lyræ variabile*, commentatio altera, p. 28); by Schur (*Ast. Nac.* No. 3283); by Markwick (*Memoirs British Astron. Assoc.* vol. xi., pt. 4) be compared and the want of complete correspondence between the light-curves of *β Lyræ* taken at wide intervals of time will be at once evident.

Yet this lack of agreement is a much more significant fact in stellar physics than any apparent and unreal agreement would be. The want of complete correspondence between the curves indicated above (of *β Lyræ*) is due to, and is therefore a measure of, the changes produced (1) by the steady recession from one another of the component stars forming *β Lyræ*; (2) by the revolution of the apsides of the system.

The former needs no proof, merely a presentment of the fact, as the regular but ever-diminishing increase of period of *β Lyræ* is one of the firmly established facts of astronomy; while the second—the revolution of the apsidal line of *β Lyræ*—is made evident by a critical examination of the various light-curves.

Yet while the deeper problems that gather round these remarkable stars can only be dealt with fully when we have a long and continuous series of observations to work upon, some of the more salient points of figure and density are readily deducible from a single light-curve, obtained even from a few but well-placed observations.

I think it will be admitted that two outstanding facts of figure and density are (1) that there is a certain definite correspondence between prolateness and distance, and (2) that the density of all close binary stars is small.

Every addition to the number of close binary stars confirms these two conclusions.

The last star added to the list, *V Vulpeculæ*, also raises two points of extreme interest in any investigation dealing with stellar evolution; and since these two matters can be satisfactorily considered from a single determination of the light-curve of the star, I think the present not unfavourable for an inquiry into their meaning and force.

The light-curve that we use as *datum* for our conclusions was deduced by Mr. Stanley Williams from observations made by himself.

The light-curve, and certain interesting matter concerning it, finds a place in the *Journ. British Astron. Assoc.* vol. xv. p. 200.

The light-curve yields the following elements of variation of *V Vulpeculæ*:

Period	75·3 Days
Mag. at max.	8·21
Mag. at prin. min.	9·65
Mag. at sec. min.	8·66
Duration of prin. eclipse...	22 Days
Duration of sec. eclipse	18 „
Duration of non-eclipse period	35 „

We use the term "non-eclipse" period instead of "constant" period for a reason that will appear later on.

In order that the proof of the prolateness of the two stars that go to make up the binary system *V Vulpeculæ* may be as convincing as possible, let it be assumed that they are spheres.

From the conditions of the problem, it is evident that during the period of non-eclipse we see both stars; that is, 8·21 mag. represents the combined light of both components, V_1 and V_2. Let this quantity of light be regarded as equal to unity.

During principal minimum V_2 eclipses V_1, causing a decrease in brightness of 1·44 mag.; that is, the light falls from unity to 0·26.

At the secondary minimum, V_1 eclipses V_2, causing a decrease of 0·45 mag., or a fall from unity to 0·66.

We have accordingly the following relations:

$$\begin{aligned} L_1 + L_2 &= 1{\cdot}00 \\ L_1 + mL_2 &= 0{\cdot}66 \\ L_2 + nL_1 &= 0{\cdot}26 \end{aligned}$$

where L_1 and L_2 represent the light of the components V_1 and V_2, and m and n are positive proper fractions, or zero. They represent the uneclipsed portions of V_2 and V_1.

From the above equations there results the identity

$$m = \frac{8 + 66n}{100n - 26}$$

It is evident that any fractional values of n make m greater than unity or a negative quantity—an impossible condition.

The two stars, therefore, which compose *V Vulpeculæ* cannot be spherical.

This conclusion is further borne out by the bow-shaped form of the curve during non-eclipse period. If the component stars were spheres the light-curve during the time of non-eclipse would be a *straight line*. The arched form, however, indicates a changing projected surface, such as that of a prolate spheroid revolving round its minor axis.

We now proceed to deal with the amount of this prolateness.

The equations that connect light-variation with the orbital elements of a close binary system are given in *Monthly Notices*, vol. lxiii. pp. 532–534, and so need not be stated again.

Put briefly the relations are, in the case of *V Vulpeculæ* :

$$1{\cdot}00 = L_1 + L_2$$
$$0{\cdot}66 = L_1 \sqrt{1 - \cos^2 \iota \varepsilon^2}$$
$$0{\cdot}26 = L_2 \sqrt{1 - \cos^2 \iota \varepsilon^2}$$

Regarding now the eclipse as a central one, that is, $\cos^2 \iota = 1$, there result the following values :

$$L_1 = 0{\cdot}72$$
$$L_2 = 0{\cdot}28$$
$$\varepsilon = 0{\cdot}39$$

The quantity ε is the eccentricity or prolateness of the figure of the component stars of the system ; that is, *V Vulpeculæ* is composed of two nearly equal masses, one of which is two and a half times brighter than the other.

The prolateness or eccentricity of figure is 0·39.

The duration of eclipse when compared with the full period of revolution yields the relative size of the component stars.

Without going into the numerical operations we find that, taking the radius of the orbit of the system as unity, the major radius of either star is 0·38 ; that is, the gap between the stars forming the system of *V Vulpeculæ* is only 0·24 in width, the radius of the orbit being unity.

These results when related to similar determinations already obtained are, we venture to think, of some importance in the study of the evolution of binary systems.

In *Monthly Notices*, vol. lxiii. p. 527, I carried out a rigorous determination of the figure and density of the close binary star *RR Centauri*, finding for this system a prolateness of 0·78 and a

distance between the components of −0·01, that is, they were still in contact.

In the *Astrophysical Journal*, vol. vii. p. 1, Mr. Myers deals with the binary system *β Lyræ*, finding a prolateness of 0·56, and that the stars had just separated.

During the past four months I have had under consideration, on the lines of the present investigation, the variation of *β Lyræ*. I find from a discussion of all available observations a mean eccentricity of figure equal to 0·58, and a mean distance (for 1850) equal to 0·01.

Placing these results in tabulated sequence we have :

System.	Distance between Components.	Prolateness according to	
		Observation.	Theory.
RR Centauri	−0·01	0·78	0·66
β Lyræ	+0·01	0·57	0·58
V Vulpeculæ	+0·24	0·37	0·35

The conclusion here is evident and unmistakable. *The nearer the components are to one another the greater their prolateness, and the amount of prolateness is in fair conformity with what theory alone would indicate.*

The density of *V Vulpeculæ* raises another and perhaps more important issue.

Without going into the numerical operations involved in arriving at the result, it may be stated that the density of the two stars which compose this remarkable binary system cannot be higher than 0·00001.

As is well known Mr. Myers found for the system *β Lyræ* a density of 0·0006.

The question arises at once : Is it possible for bodies so tenuous to hold together as they sweep round one another in rapid revolution ? The question is a legitimate one, but it is beyond the scope of the present paper to deal with it.

My object rather has been to bring forward the variation of *V Vulpeculæ* as proof of a certain definite law of prolateness in close binary systems, and also as proof that such stars are composed of matter of extreme rarity of structure.

New Double Stars. By the Rev. T. E. Espin, M.A.

The following list contains all the new pairs detected since the end of 1902. A large number of them were found during the autumn of 1904 and the spring of the present year, and consequently were too late for insertion in Professor Burnham's new catalogue. In many cases these pairs are too difficult to measure properly with the means at my disposal. Four wide